

# Optimal reservoir operation policies using novel nested algorithms

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# Introduction

- Optimal reservoir operation methods include:
  - Dynamic programming (DP)
  - Stochastic dynamic programming (SDP)

**“curse of dimensionality”**

the computational complexity with the  
state – decision space dimension

Successive approximations, incremental  
dynamic programming and differential  
dynamic programming

**“curse of modelling”**

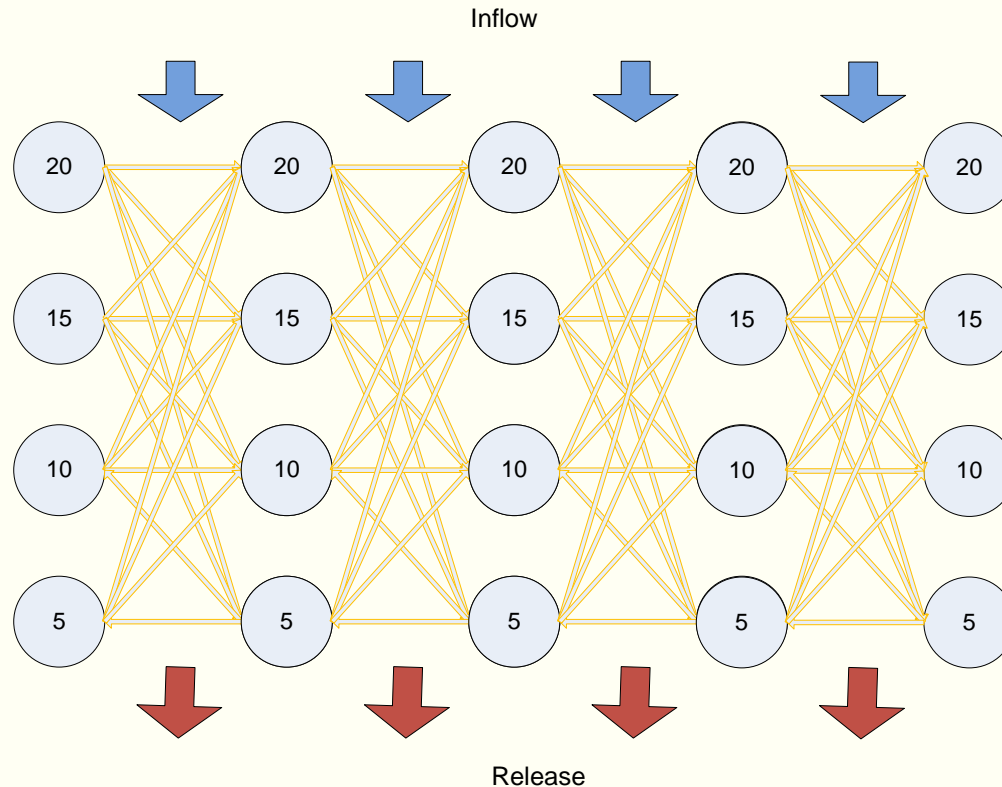
# Dynamic programming

$$x_{t+1} = x_t + q_t - r_t - e_t$$

Reservoir model

$$V_t(s_t) = \min \{ g(s_t, s_{t+1}, a_t) + V_{t+1}(s_{t+1}) \} \quad (\text{for stages } t=T-1, T-2, \dots, 1)$$

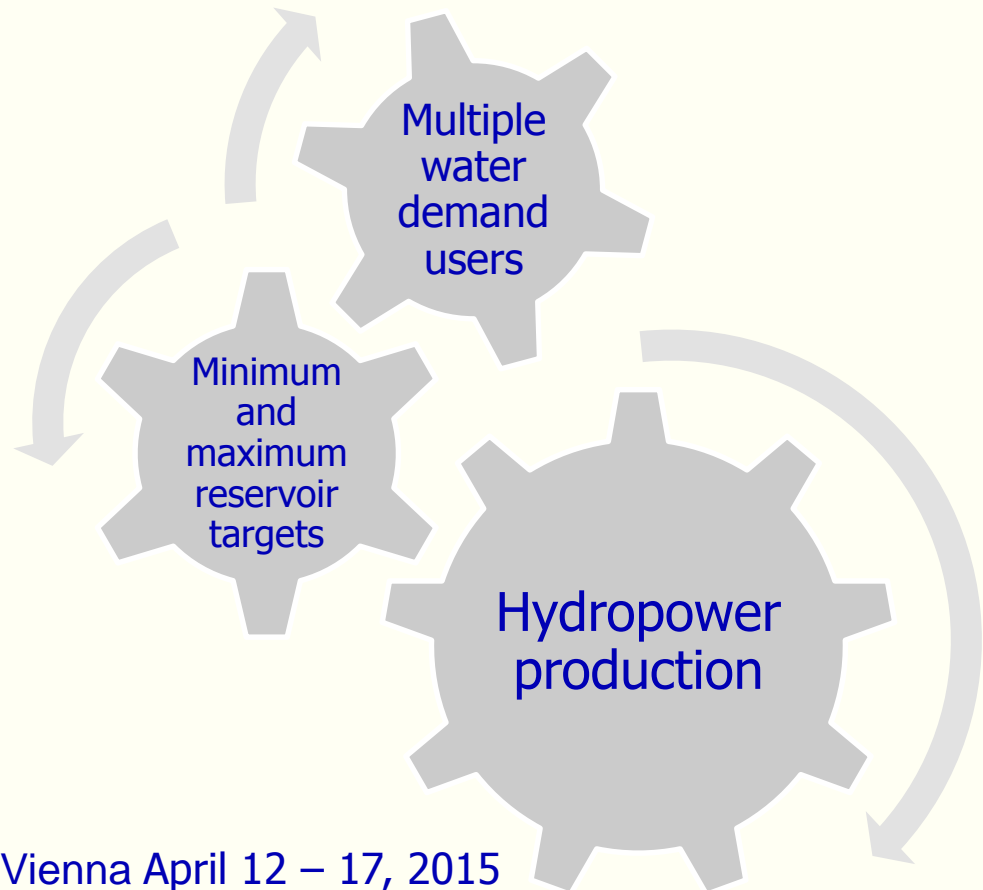
Bellman equation



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# Main question

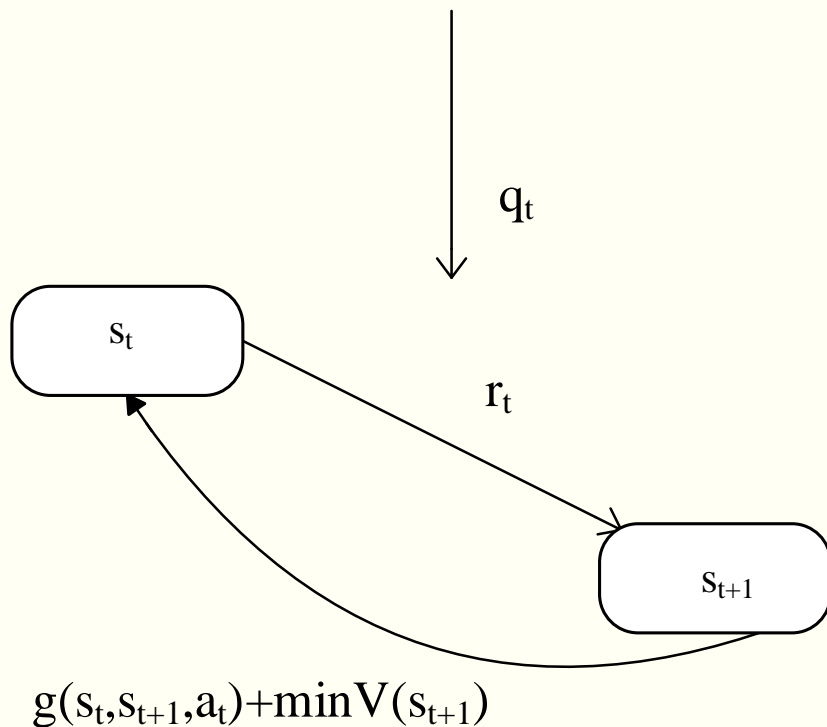
How to enhance DP/SDP algorithm and develop new algorithm (methodology) that is flexible with including additional objectives like cities water demand, agriculture water demand, ecology water demand, hydro power production and etc., alleviate as much as possible the curse of dimensionality and computational cost?



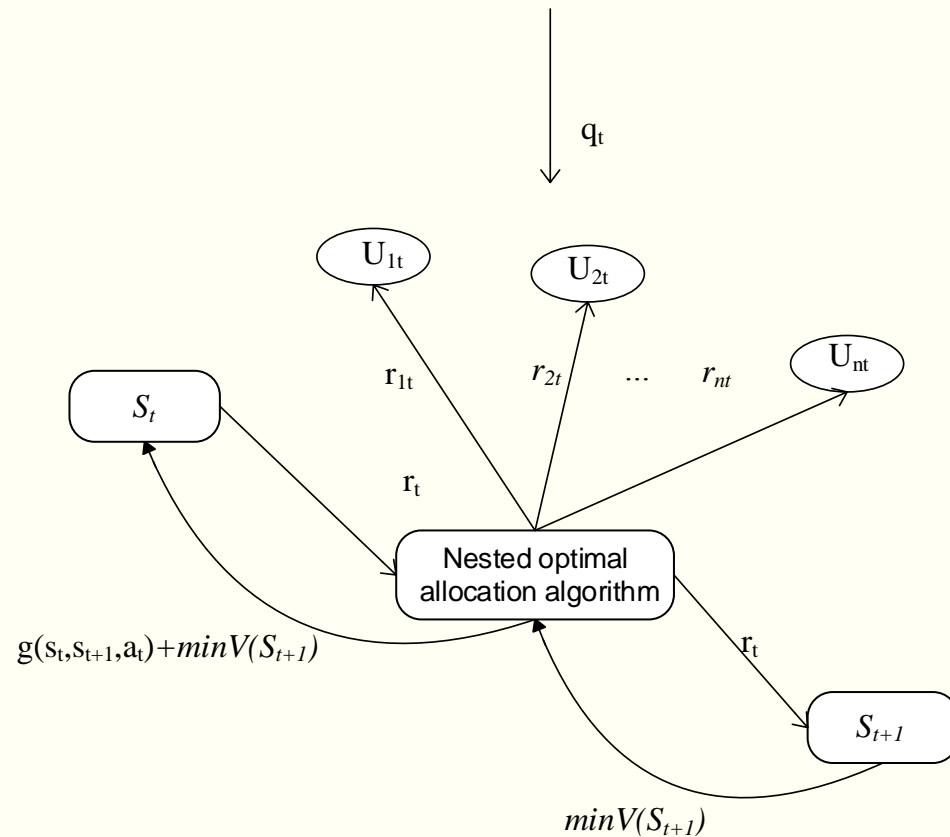
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# Main idea nested algorithm

One step of the DP algorithm.



One step of the nDP algorithm



# Nested optimal allocation algorithms

## Simplex – Quadratic Knapsack

$$r_{1t} + r_{2t} \dots + r_{nt} \leq r_t$$

$$r_{1t} \leq d_{1t}, r_{2t} \leq d_{2t} \dots, r_{nt} \leq d_{nt}$$

$$r_t, d_{t1}, d_{t2} \dots, d_{nt} \geq 0$$

$$\min \sum_{i=1}^n w_{it} \cdot (d_{it} - r_{it})$$

Simplex

$$\min \sum_{i=1}^n w_{it} \cdot (d_{it} - r_{it})^2$$

Quadratic Knapsack

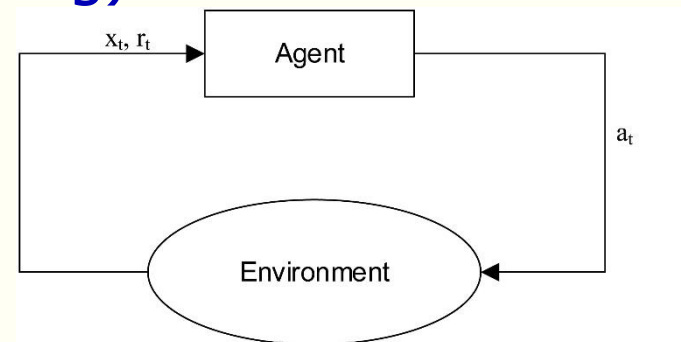
# Implementing the idea into nSDP and nRL

- The same idea of nesting optimization algorithm is implemented in nSDP (nested Stochastic dynamic programming)

$$V_t(s_t) = \min \left\{ g(s_t, s_{t+1}, a_t) + \gamma^t \cdot \sum_j p_{q_{t+1}|q_t} \cdot V_{t+1}(s_{t+1}) \right\}$$

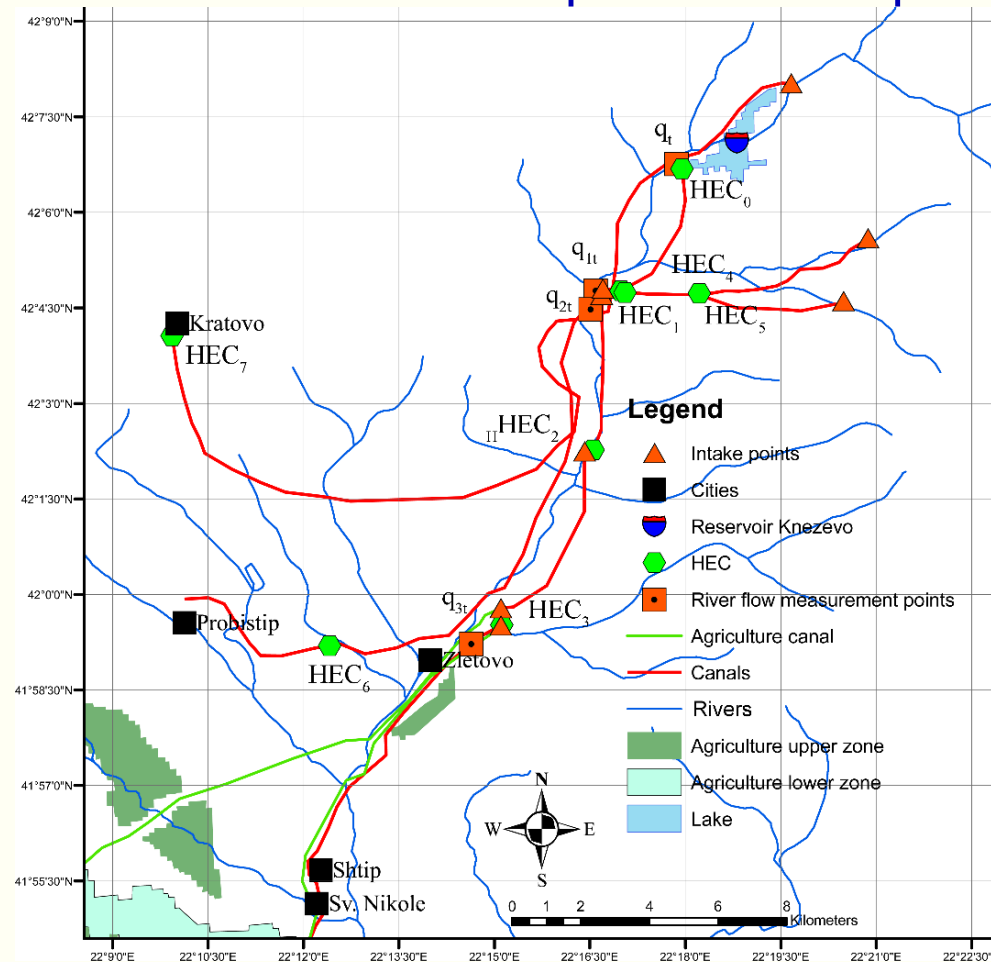
$$p_{q_{t+1}|q_t} = P\{q_{t+1} \text{ in interval } t+1 \mid q_t \text{ in interval } t\}$$

and nRL (nested Reinforcement learning)



# Zletovica river basin

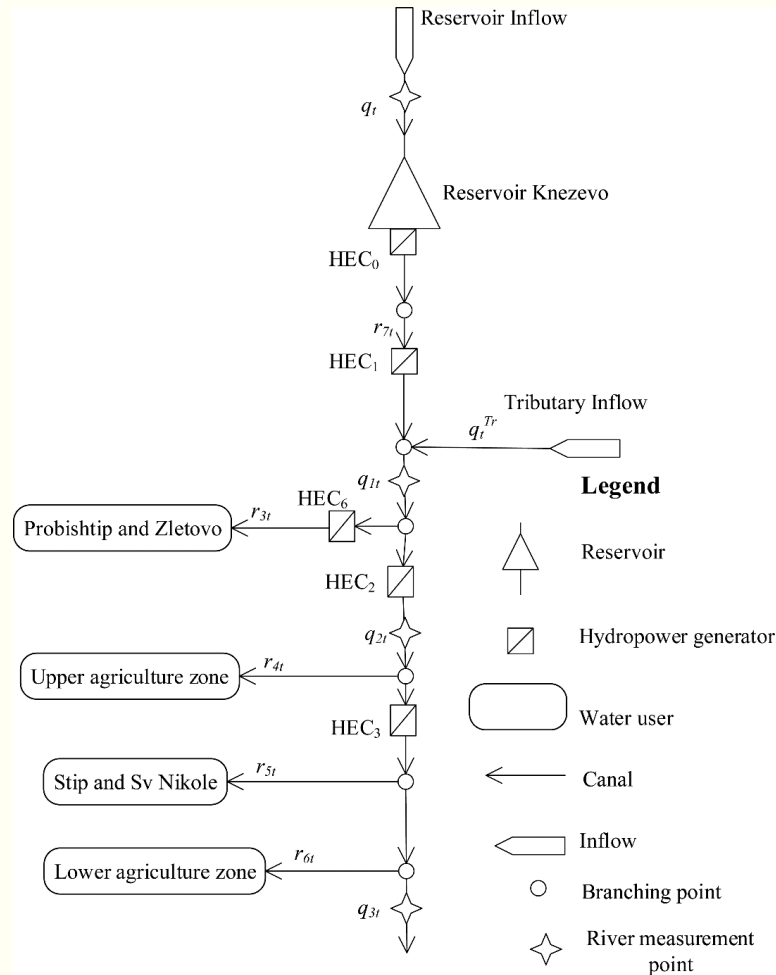
Zletovica river basin is in the eastern part of the Republic of Macedonia.



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# Schematic representation of Zletovica river basin



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# Objective function

The optimization problem has eight objectives and six decision variables.

weighted sum of squared deviations over the entire time horizon

$$g_t(s_t, s_{t+1}, a_t) = \sum_{i=1}^8 w_{it} \cdot D_{it}^2$$

$$D_{1t} = \begin{cases} 0, & \text{if } d_{1t} \geq h_t \\ h_t - d_{1t}, & \text{if } d_{1t} < h_t \end{cases}$$

$$D_{2t} = \begin{cases} 0, & \text{if } d_{2t} \leq h_t \\ d_{2t} - h_t, & \text{if } d_{2t} > h_t \end{cases}$$

where  $d_{1t}$  and  $d_{2t}$  are the minimum and maximum level targets and  $h_t$  is the reservoir level height at time step  $t$

# Objective function

- Water demand users

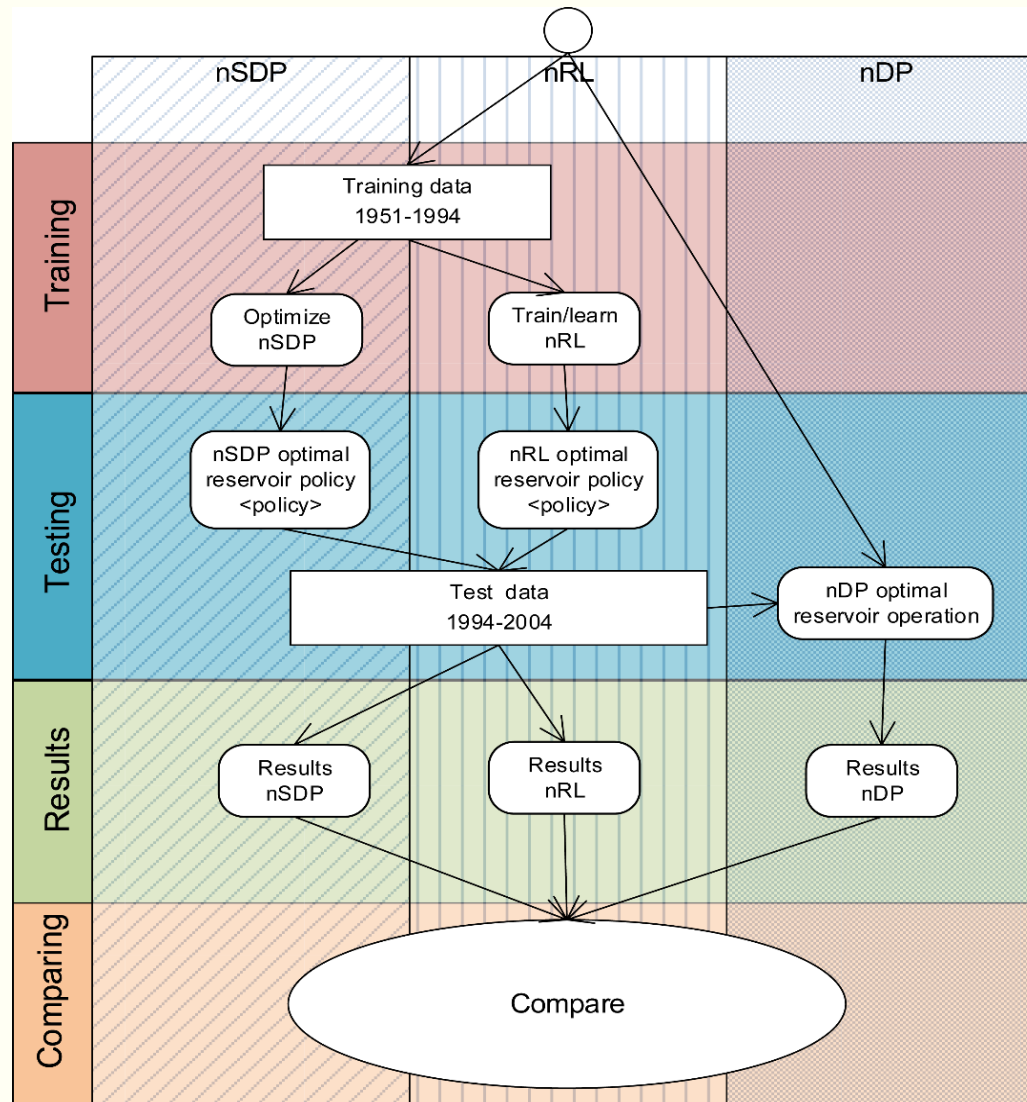
- 1) the towns of Zletovo and Probishtip (one intake),  $d_{3t}$
- 2) the upper agricultural zone,  $d_{4t}$
- 3) the towns of Shtip and Sveti Nikole (one intake),  $d_{5t}$
- 4) the lower agricultural zone,  $d_{6t}$
- 5) the minimum environmental flow,  $d_{7t}$

$$D_{it} = \begin{cases} 0, & \text{if } d_{it} \leq r_{it} \\ d_{it} - r_{it}, & \text{if } d_{it} > r_{it} \end{cases}$$

where  $h_t$  is the hydropower demand and  $p_t$  is the hydropower production

$$D_{8t} = \begin{cases} 0, & \text{if } d_{8t} \leq p_t \\ d_{8t} - p_t, & \text{if } d_{8t} > p_t \end{cases}$$

# Comparing optimal reservoir policies



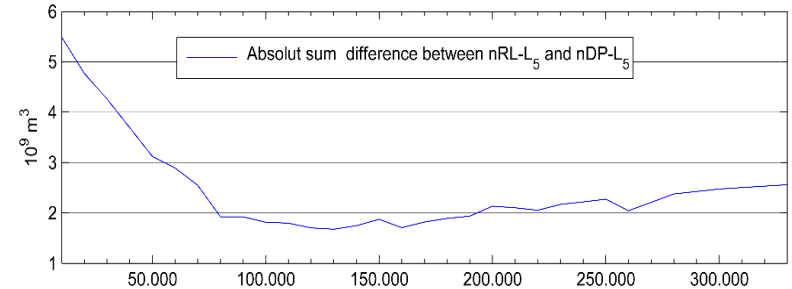
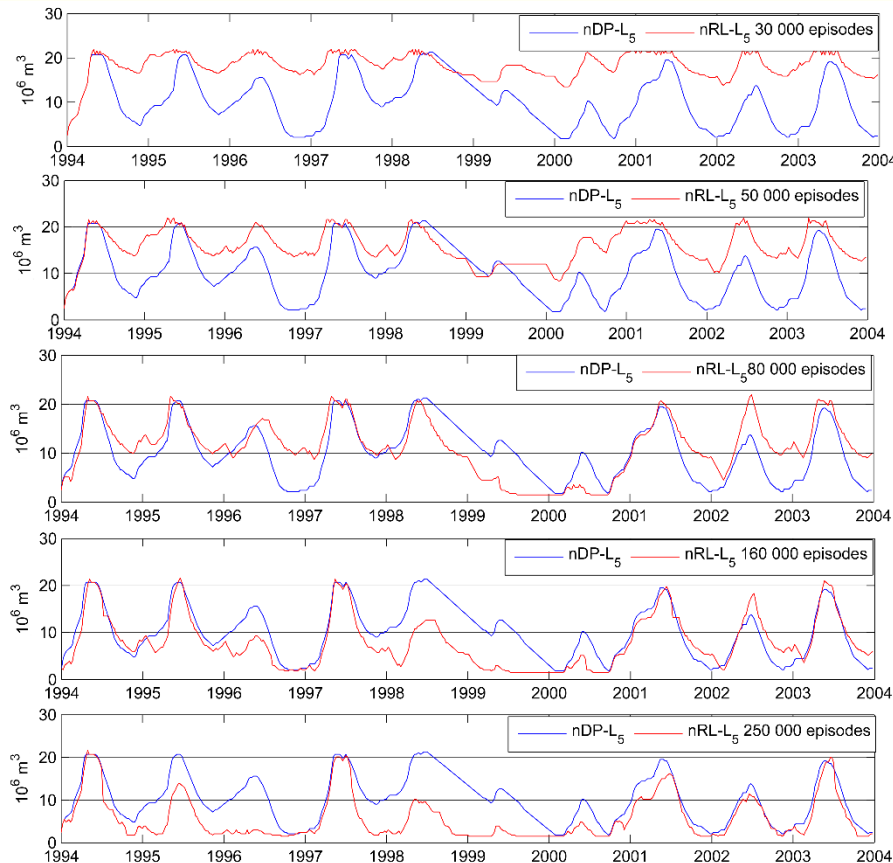
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# Results

- 55 years weekly data is separated in two parts 1) training and 2) testing. The data from 1951-1994 (2340 time-steps) is used for training and 1994-2004 (520 time-steps) for testing
- The reservoir operation volume 23 million m<sup>3</sup> is discretized in 73 equal levels (300 10<sup>3</sup> m<sup>3</sup> each).
- The minimum reservoir level target was set at 1021.5 [amsl],  
and the maximum reservoir level target at 1060 [amsl].
- The water supply, irrigation, and hydropower are set to their weekly demands
- The nDP is executed with 10 different sets of weights to provide screening of possible multi-objective solutions in the training data.
- The optimal reservoir policy has this structure <time step, storage volume, reservoir inflow, tributary inflow, next reservoir storage> ( $\langle t, x_t, q_t, q_t^{Tr}, x_{t+1} \rangle$ )

weights	w <sub>1</sub>	w <sub>2</sub>	w <sub>3</sub>	w <sub>4</sub>	w <sub>5</sub>	w <sub>6</sub>	w <sub>7</sub>	w <sub>8</sub>
nDP-L <sub>5</sub> , nDP-Q <sub>5</sub>								
nSDP-L <sub>5</sub> , nSDP-Q <sub>5</sub>	2,000,000	2,000,000	200	1	200	1	300	0.01
nRL-L <sub>5</sub> , nRL-Q <sub>5</sub>								

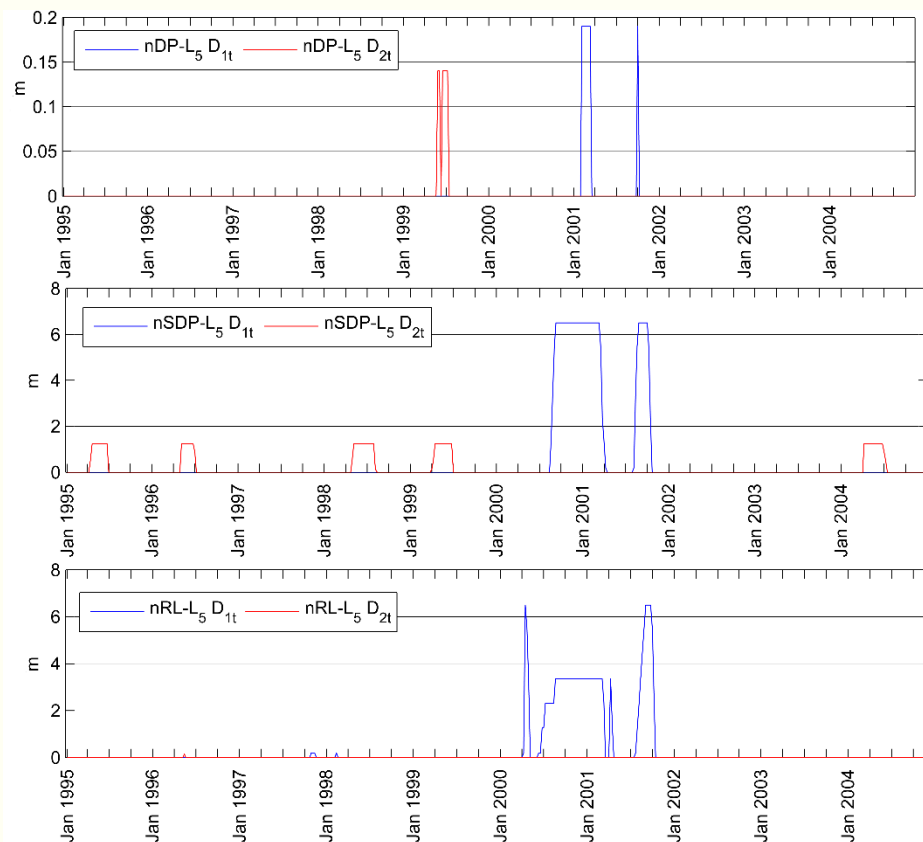
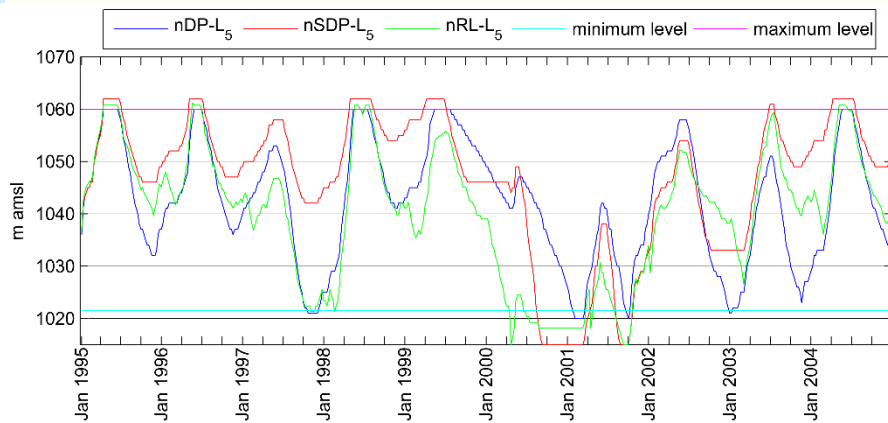
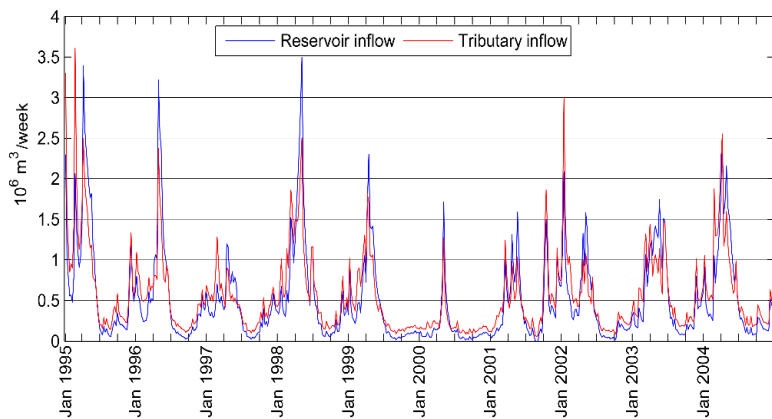
# nRL agent learning



Sum of absolute difference between nRL-L5 and nDP-L5 measured as difference in reservoir volumes in period 1994-2004 as a function of the nRL number of episodes.

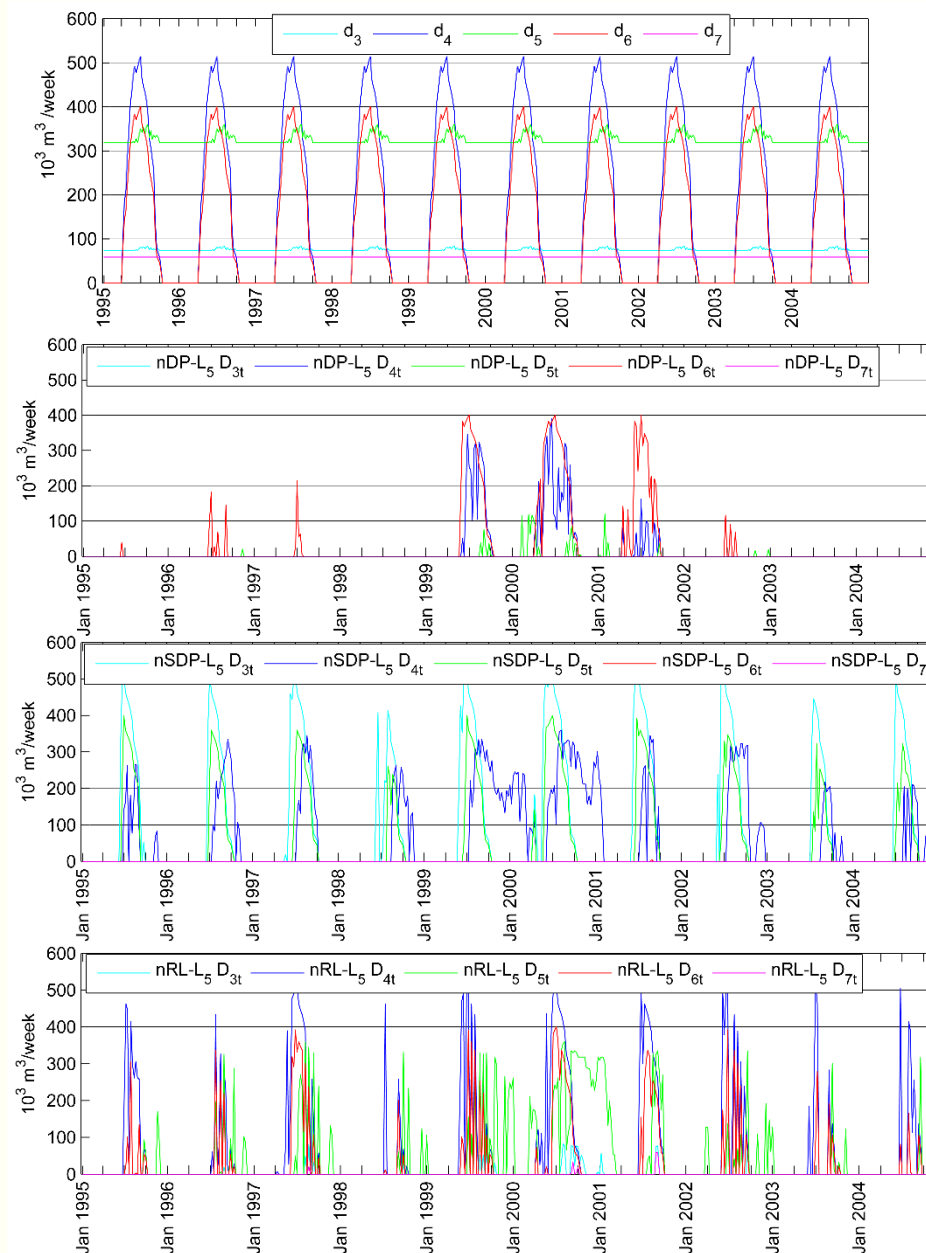
nRL-L5 agent learning with increasing the number of episodes: nDP-L5 target reservoir storage (blue) and nRL-L5 obtained reservoir storage (red) (testing period)

# Results



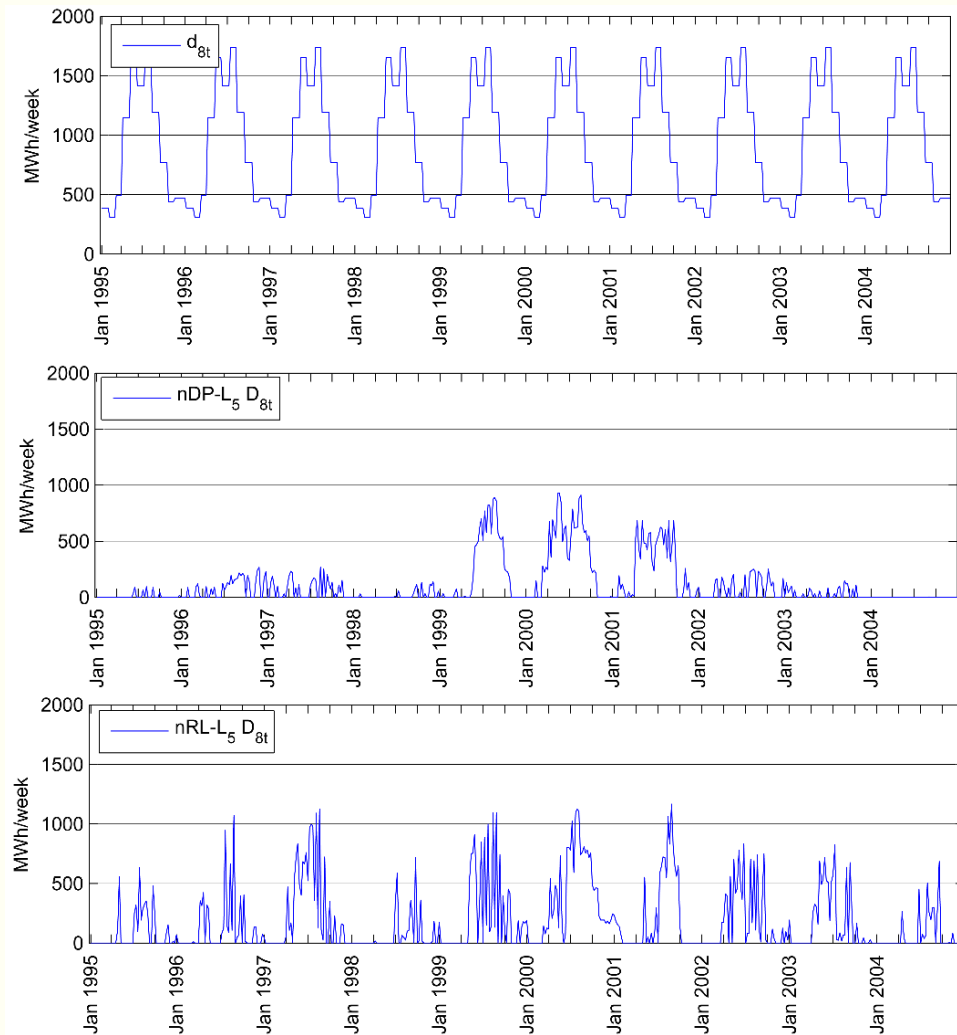
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# Results



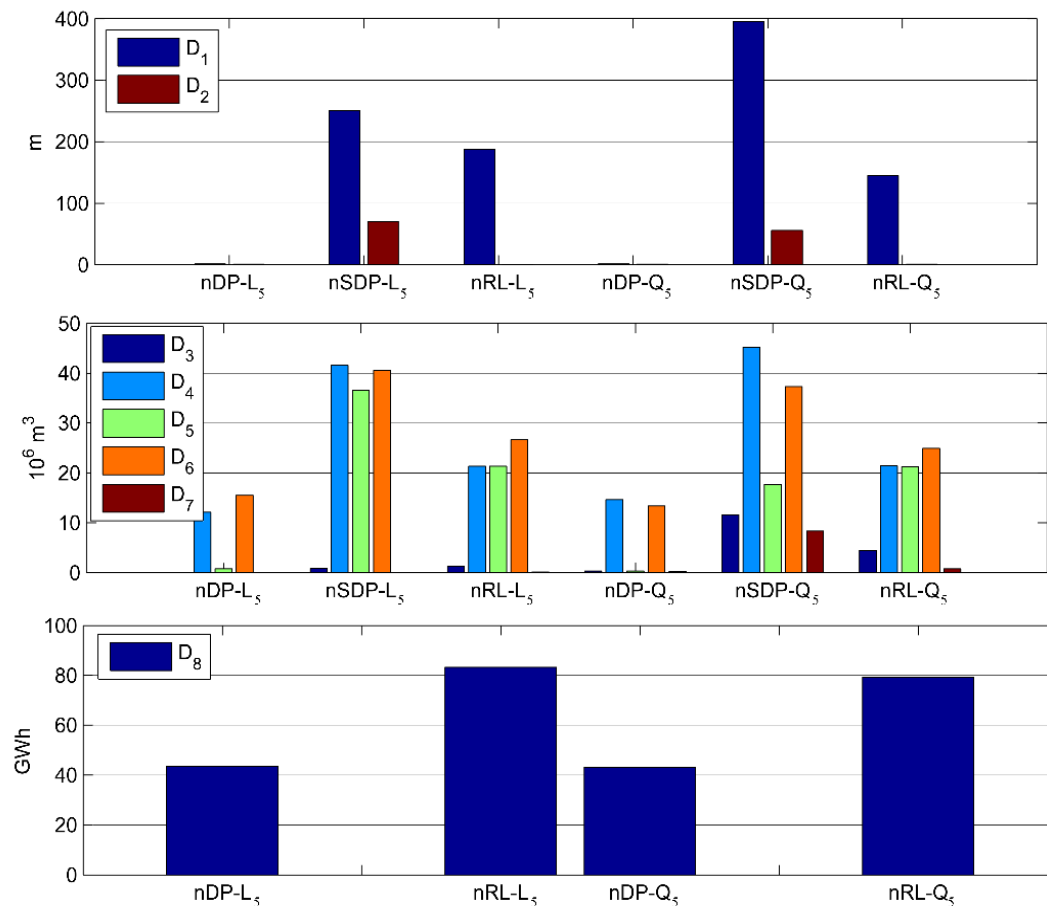


# Results



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# Results



Comparison of the sum of minimum level (D<sub>1</sub>) and maximum level (D<sub>2</sub>) deviations, sum of users' deficit (D<sub>3</sub>-D<sub>7</sub>) and sum of hydropower deficit (D<sub>8</sub>) in the testing period 1994-2004

# Conclusions

The nested algorithms have the following advantages:

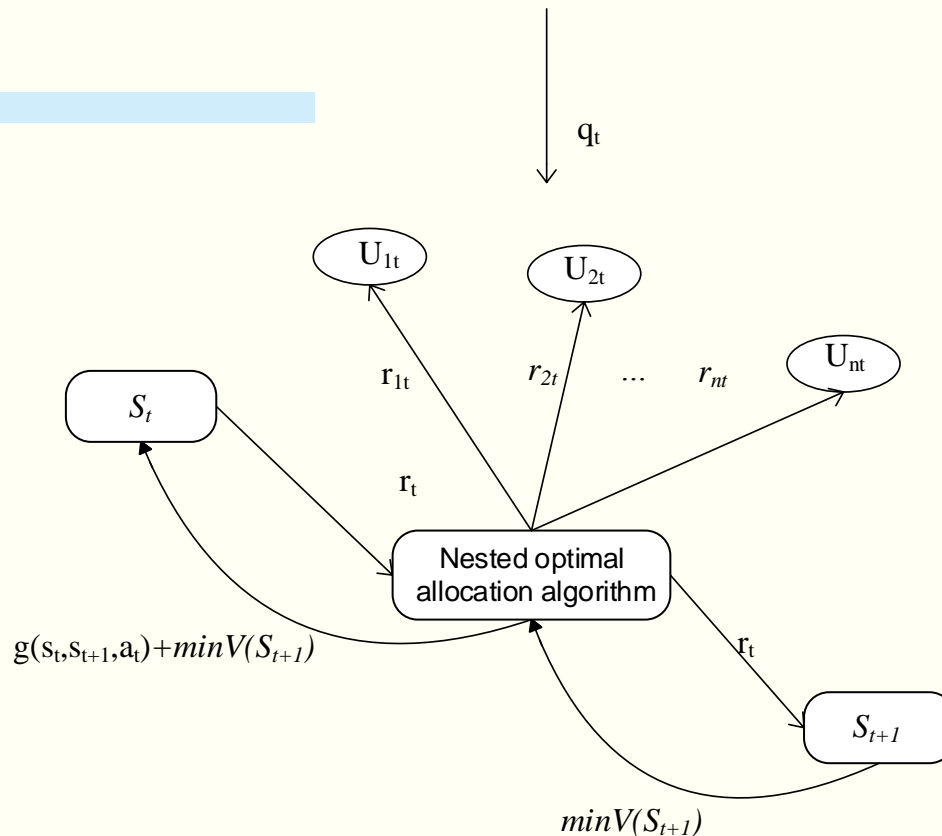
- Effectively **alleviate the curse of dimensionality** in optimal reservoir operation. (optimization problem has eight objectives and six decision variables)
- They have **better optimization capabilities** compared to the DP aggregated water demand approach (and similar SDP and RL) and can solve problems that are more complex where the DP aggregated water demand approach is not feasible.
- Computationally, they are **very efficient** and runs fast on standard personal computers. The presented case study optimization was executed in less than **five minutes**.
- The algorithms allow for **employing dense and variable discretization** on the reservoir volume and release.
- Support using a **variable weight** at each time step for every objective function.

# Conclusion

- Provides a **framework for including many objectives** in the nested optimization algorithm without a significant change in the source code or an increase in the computational expenses.
- **Different optimization algorithms** can be used in the nesting for water allocation, however, since nested optimization has to be repeated multiple times (for each transition of DP) the algorithm used for this purpose needs to be fast.
- The nSDP and nRL policies were benchmarked against the nDP results and it was obvious that the nRL performs better than nSDP overall and for all objectives separately. The main conclusion is that nRL is a better and more capable optimal reservoir algorithm than nSDP, at least in this case study.

The main idea is to include nested optimization algorithm inside the state transition, which lowers the problem dimension. With this method, it is possible to solve optimization problems that are currently unsolvable with classical methods, rapidly decrease the optimization time and improve the result that was demonstrated in this paper.

# Thank you



Delipetrev, B., Jonoski A., Solomatine D.  
(2015) "A novel nested dynamic  
programming (nDP) algorithm for  
multipurpose reservoir optimization"  
*Journal of Hydroinformatics*.  
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<https://github.com/deblagoj/>

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